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Wilson and Kadowaki-Woods ratios in heavy fermions

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Abstract. Recently we have shown that a one-parameter scaling, $T_{\rm coh}$, describes the physical behavior of several heavy fermions in a region of their phase diagram. In this paper we fully characterize this region, obtaining the uniform susceptibility, the resistivity and the specific heat in terms of the coherence temperature $T_{\rm coh}$. This allows for an explicit evaluation of the Wilson and the Kadowaki-Woods ratios in this regime. These quantities turn out to be independent of the distance $|\delta|$ to the quantum critical point (QCP). The theory of the one-parameter scaling corresponds to a local interacting model. Although spatial correlations are irrelevant in this case, time fluctuations are critically correlated as a consequence of the quantum character of the transition.

PACS. 71.27.+a Strongly correlated electron systems; heavy fermions – 75.30.Mb Valence fluctuation, Kondo lattice, and heavy-fermion phenomena – 71.10.Hf Non-Fermi-liquid ground states, electron phase diagrams and phase transitions in model systems

1 Introduction

Most of the physical properties of heavy fermions can be attributed to the proximity of these systems to a quantum critical point (QCP) [1,2]. This zero temperature critical point arises as a competition between Kondo effect and magnetic order induced by RKKY coupling. In the magnetically disordered side of the phase diagram a scaling approach reveals the existence of a new characteristic temperature, the *coherence temperature* $T_{\rm coh} \propto |\delta|^{\nu z}$, below which the system exhibits Fermi liquid (FL) behavior [3]. In this relation, $|\delta| = |J_Q - J_Q^c|$ measures the distance to the $T = 0$ critical point and ν and z are respectively the correlation length and dynamic critical exponents. J_Q is the coupling between the local moments and J_Q^c its critical value, at which the magnetic instability, characterized by the wavevector Q occurs. At the QCP, *i.e.*, $|\delta| = 0$, the system does not cross the *coherence line* and consequently exhibits *non-Fermi liquid behavior* down to $T = 0$ [1].

Recently, we have shown that the heavy fermion systems $CeRu_2Si_2$, $CeCu_6$, UPt_3 and $CeAl_3$ under pressure (the latter for $P \geq 1.2$ kbars), obey a *one-parameter scaling*, *i.e.*, $C/T \propto T_{\rm coh}^{-1}$, $\chi_0(T \rightarrow 0) \propto T_{\rm coh}^{-1}$, $A_R \propto T_{\rm coh}^{-2}$ and $h_{\rm c} \propto T_{\rm coh}$, where these quantities are measured in the Fermi liquid regime for $T \ll T_{\rm coh}$ [1]. The pressure dependence of the coherence temperature, $T_{\rm coh}$, then determines the pressure variation of the coefficient of the linear term of the specific heat, of the uniform susceptibility, the coefficient of the T^2 term of the resistivity and of the characteristic pseudo-metamagnetic field, respectively. In this paper we use the spin-fluctuation theory of a nearly anti-ferromagnetic metal [5,6] to fully characterize this regime and calculate the specific heat, the uniform susceptibility and the resistivity in the region of the phase diagram where this type of one-parameter scaling is observed. This allows for an explicit evaluation of the *Wilson ratio* and the *Kadowaki-Woods ratio* [7,8] between the coefficient of the T^2 term in the resistivity and that of the linear term of the specific heat.

It is important to emphasize that, although the spin fluctuation model that we use to describe the physical behavior associated with the antiferromagnetic quantum critical point is a Gaussian theory, this theory gives the *exact description* of the quantum critical behavior [1], *i.e.*, the Gaussian exponents associated with the QCP are exact. The reason is that for the problem considered here, the effective dimension $d_{\rm eff} = d + z$. Since the Euclidean dimension $d = 3$ and the dynamic exponent $z = 2$, for antiferromagnetic fluctuations, $d_{\rm eff} > d_{\rm c} = 4$, the upper critical dimension for the magnetic transition. Consequently the Gaussian fixed point yields the correct zero temperature critical exponents [9].

2 Specific heat

We start from the expression for the free energy given by the spin-fluctuation theory of a nearly antiferromagnetic electronic system [5,6]. We use the notation of reference [6],

$$f_{\rm sf} = -\frac{3}{\pi} \sum_{\mathbf{q}} T \int_0^\infty \frac{{\rm d}\lambda}{{\rm e}^\lambda - 1} \tan^{-1}\left(\frac{\lambda T}{\Gamma_q}\right) \qquad (1)$$

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where

$$\Gamma_q = \Gamma_{\rm L}(1 - J_Q\chi_{\rm L}) + \Gamma_{\rm L}\chi_{\rm L}Aq^2.$$

$\Gamma_{\rm L}$ and $\chi_{\rm L}$ are local parameters defined through the local dynamical susceptibility, $\chi_{\rm L}(\omega) = \chi_{\rm L}/(1-{\rm i}\omega/\Gamma_{\rm L})$ [6]. J_Q, as before, is the q-dependent exchange coupling between f-moments and A is the *stiffness* of the lifetime of the spin fluctuations defined by the small wavevector expansion of the magnetic coupling close to the wavevector Q, *i.e.*, $J_Q - J_{Q+q} = Aq^2 + \cdots$. Then, Γ_q can be rewritten as, $\Gamma_q = \Gamma_{\rm L}\chi_{\rm L}A\xi^{-2}[1 + q^2\xi^2]$, where the correlation length, $\xi = \left(A/|J_Q - J_Q^{\rm c}|\right)^{1/2} = \sqrt{A/|\delta|}$ diverges at the critical value of the coupling, $J_Q^{\rm c} = \chi_{\rm L}^{-1}$, with the Gaussian critical exponent, $\nu = 1/2$. Consequently, we have for the specific heat $C/T = -\frac{\partial^2 f_{\rm sf}}{\partial T^2}$,

$$C/T = \frac{\partial^2}{\partial T^2}\left\{\frac{3}{\pi}\sum_{\mathbf{q}} T\int_0^\infty \frac{d\lambda}{{\rm e}^\lambda - 1}\tan^{-1}\right. \\ \left.\times\left(\frac{\lambda T\xi^z}{\Gamma_{\rm L}\chi_{\rm L}A(1+q^2\xi^2)}\right)\right\} \qquad (2)$$

where we have identified the dynamic critical exponent, $z = 2$, typical of antiferromagnetic spin fluctuations [5]. The argument of the $\tan^{-1}$ can be written as, $(\lambda T\xi^z/A\Gamma_{\rm L}\chi_{\rm L})[1 - q^2\xi^2/(1+q^2\xi^2)]$. The term in brackets [...] is always ≤ 1, furthermore the exponential cuts off the contribution for the integral from large values of λ, consequently for $(T\xi^z/\Gamma_{\rm L}\chi_{\rm L}A) \ll 1$, we can expand the $\tan^{-1}$ for small values of its arguments. It is easily seen that this condition can be written as, $T \ll T_{\rm coh}$, where the *coherence temperature*,

$$k_{\rm B}T_{\rm coh} = \Gamma_{\rm L}\chi_{\rm L}|J_Q - J_Q^{\rm c}| = \Gamma_{\rm L}\chi_{\rm L}|\delta|^{\nu z}$$

in agreement with the scaling theory, since $\nu z = 1$ ($\nu = 1/2$ and $z = 2$) [1]. Notice that $T_{\rm coh}$ is *independent* of A.

Then we have for $T \ll T_{\rm coh}$,

$$C/T = \frac{\partial^2}{\partial T^2}\left[\frac{3T^2}{\pi T_{\rm coh}}\int_0^\infty \frac{{\rm d}\lambda\lambda}{{\rm e}^\lambda - 1}\sum_{\mathbf{q}}\frac{1}{1+q^2\xi^2}\right]\cdot \qquad (3)$$

This equation clarifies the physical meaning of the coherence temperature. It is the characteristic temperature, below which, the free energy is quadratic in temperature and consequently the system exhibits Fermi liquid behavior. Changing the $\sum_{\mathbf{q}}$ into an integral we find ($d = 3$),

$$C/T = \frac{\partial^2}{\partial T^2}\left[\frac{\pi T^2\xi^{(z-d)}}{2\Gamma_{\rm L}\chi_{\rm L}A}\frac{4\pi V}{(2\pi)^3}\int_0^{q_{\rm c}\xi}\frac{{\rm d}yy^2}{1+y^2}\right] \qquad (4)$$

which yields

$$C/T = \frac{\pi\xi^{(z-d)}}{\Gamma_{\rm L}\chi_{\rm L}A}\frac{4\pi V}{(2\pi)^3}q_{\rm c}\xi\left(1 - \frac{\tan^{-1}q_{\rm c}\xi}{q_{\rm c}\xi}\right)\cdot \qquad (5)$$

In the critical regime, $q_{\rm c}\xi \gg 1$, we obtain the result of reference [6], *i.e.*, $C/T = 6\pi^2Nk_{\rm B}^2/Aq_{\rm c}^2$, a non-universal, cutoff dependent value. In the opposite, sub-critical regime, *i.e.*, for, $q_{\rm c}\xi \ll 1$, since $\tan^{-1}y \approx y - y^3/3 + y^5/5 + \cdots$ for small y, we get [1]

$$C/T = \frac{\pi\xi^{(z-d)}}{\Gamma_{\rm L}\chi_{\rm L}A}\frac{4\pi V}{(2\pi)^3}q_{\rm c}\xi\left[\frac{1}{3}(q_{\rm c}\xi)^2 - \frac{1}{5}(q_{\rm c}\xi)^4 + \cdots\right]\cdot \qquad (6)$$

The first term is independent of A and yields,

$$C/T = \frac{\pi Nk_{\rm B}^2}{\Gamma_{\rm L}\chi_{\rm L}}\frac{1}{|J_Q - J_Q^{\rm c}|} = \frac{\pi Nk_{\rm B}}{T_{\rm coh}}\cdot \qquad (7)$$

In fact this could have been obtained directly, from equation (3), neglecting the q-dependence of Γ_q and with $\sum_{\mathbf{q}} \to N$ [10]. In the equation above the correct units have been restored. In principle, one may think naively that this limit is not relevant, since the condition, $q_{\rm c}\xi \ll 1$, can only be satisfied far away from the quantum critical point. Note however, that this may be written as, $q_{\rm c}\sqrt{A/|\delta|} \ll 1$, which can be satisfied, *either because the system is far away from the quantum critical point, i.e.,* $|\delta|$ *is large, or because* A *is small.* Rewriting this condition as $q_{\rm c}/\sqrt{|\delta|} \ll 1/\sqrt{A}$, we notice that, when $A \to 0$, it holds *arbitrarily close* to the QCP, *i.e.*, for δ arbitrarily small.

Table 1 gives values for the coherence temperature for some heavy fermion systems, which obey one parameter scaling, obtained from equation (7) and the measured specific heat.

3 Susceptibility and Wilson ratio

The zero temperature uniform susceptibility of the nearly antiferromagnetic system in the limit $q_{\rm c}\xi \ll 1$ can be directly obtained from the magnetic field (h) dependent, $T = 0$, q-independent free energy [1,12],

$$f_{\rm sf}^{\rm L} = -\frac{3N}{2\pi}\int_0^{\omega_{\rm c}}{\rm d}\omega\tan^{-1}\left[\frac{\omega + \mu h}{\Gamma_{\rm L}\chi_{\rm L}|J_Q - J_Q^{\rm c}|}\right]\cdot \qquad (8)$$

Notice that the magnetic field in this expression scales as $(h/h_{\rm c})$ where the characteristic field, $h_{\rm c} \propto T_{\rm coh}$. The uniform susceptibility χ_0 is given by,

$$\chi_0 = -\left(\frac{\partial^2 f_{\rm sf}^{\rm L}}{\partial h^2}\right)_{h=0} = \frac{3N\mu^2}{2\pi\Gamma_{\rm L}\chi_{\rm L}}\frac{1}{|J_Q - J_Q^{\rm c}|} = \frac{3N\mu^2}{2\pi k_{\rm B}T_{\rm coh}} \qquad (9)$$

where the limit $\omega_{\rm c} \to \infty$ has been taken. Note from the general expression for the uniform susceptibility [6], $\chi^{-1} = \chi_Q^{-1} + Aq_{\rm c}^2$, where χ_Q is the staggered susceptibility, that in the limit, $q_{\rm c}\xi \ll 1$, $\chi = \chi_Q = \chi_0$. Then the staggered and uniform susceptibilities coincide in this limit, being equally enhanced.

Table 1. Parameters for some heavy fermion systems. All experimental data are taken from [11] and references therein. (*) obtained from equation (7) for the specific heat. (**) along the c-axis. (***) in the basal plane. For the transport parameter, the average number of conduction electrons per atom, $n = 1$.

	$CeRu_2Si_2$	$CeCu_6$	UPt_3
$\mu(\mu_B)$	2.5	2.5	3.0
$v_0^{1/3}(\times 10^{-8}$ cm$)$	4.4	4.7	4.1
T_{coh} (K)*	67	15	58
χ/μ^2 $(\times 10^{35}$ erg^{-1}cm$^{-3})$	5.8	7.9	2.5
$\frac{C/T}{\pi^2 k_B^2}$ $(\times 10^{35}$ erg^{-1}cm$^{-3})$	3.9	13.0	5.6
$\frac{\chi/\mu^2}{C/T\pi^2 k_B^2}$	1.46	0.59	0.44
A_R $(\mu\Omega$ cm K$^{-2})$	0.40	14.4	0.7**–1.6***
$\frac{A_R}{(C/T)^2}$ $\left(\times 10^{-5}\mu\Omega \text{ cm}\left(\frac{\text{mol K}}{\text{mJ}}\right)^2\right)$	0.27	0.51	0.34**–0.79***
$(\Gamma_L\chi_L)(J/W)^2$	0.40	0.71	0.54**–1.25***
(J/W)	0.47	0.63	0.39**–0.59***

The Wilson ratio (WR) is given by

$$\frac{\chi_0/\mu^2}{C/\pi^2 k_B^2 T} = \frac{3}{2} = 1.5$$

which turns out to be a universal number since the dependence on the distance to the critical point, $|J_Q - J_Q^c|$ and on the dimensionless quantity $\Gamma_L\chi_L$ cancels out. We emphasize that the above result for χ_0 is valid in the regime, $q_c\xi \ll 1$, that is, if the system satisfies the condition, $q_c/\sqrt{|J_Q - J_Q^c|} \ll 1/\sqrt{A}$. The experimental value for the WR in $CeRu_2Si_2$, given in Table 1, is in fair agreement with the theory. The same is not true for the other systems. Since there is a reasonable spread on the experimental data for different samples of these materials, at this point, the best indication that they are in the subcritical regime is the fact that they obey one-parameter scaling.

4 Resistivity and Kadowaki-Woods ratio

The resistivity due to spin fluctuations in the regime $q_c\xi \ll 1$ is given by [1,13]

$$\rho = \rho_0 \frac{1}{T}\int_0^\infty d\omega \frac{\omega \Im m\chi_Q(\omega)}{(e^{\beta\omega}-1)(1-e^{-\beta\omega})} \qquad (10)$$

where

$$\Im m\chi_Q(\omega) = \chi_Q^s \frac{\omega\xi_L^z}{1+(\omega\xi_L^z)^2} \qquad (11)$$

with

$$\chi_Q^s = \frac{1}{|J_Q - J_Q^c|}$$

and

$$\xi_L^z = \frac{\chi_Q^s}{\Gamma_L\chi_L}$$

The quantity ρ_0 is given by,

$$\rho_0 = \left(\frac{J}{W}\right)^2 \frac{m_c}{N_c e^2 \tau_{Fc}}(N/N_c)$$

where J is the coupling constant per unit cell between localized and conduction electrons. W, m_c and N_c are the bandwidth, the mass and the number of conduction electrons per unit volume with Fermi momentum k_{Fc}, such that, $\hbar\tau_{Fc}{}^{-1} = \hbar^2 k_{Fc}^2/2m_c$ and N is the number of atoms per unit volume.

Using the definitions above, we can rewrite the spin-fluctuation resistivity as, $\rho = \rho_0\Gamma_L\chi_L R(\tilde{T})$, where

$$R(\tilde{T}) = \frac{1}{\tilde{T}}\int_0^\infty d\tilde{\omega}\frac{1}{(e^{\tilde{\omega}/\tilde{T}}-1)(1-e^{-\tilde{\omega}/\tilde{T}})}\frac{\tilde{\omega}^2}{1+\tilde{\omega}^2} \qquad (12)$$

with $\tilde{\omega} = \omega\xi_L^z$ and $\tilde{T} = T\xi_L^z$. For, $T \ll T_{coh} = (\Gamma_L\chi_L/k_B)|J_Q - J_Q^c|$, we have, $R(T \ll T_{coh}) \approx \frac{\pi^2}{3}\left(\frac{T}{T_{coh}}\right)^2$ and finally,

$$\rho(T \ll T_{coh}) = \rho_0\Gamma_L\chi_L\frac{\pi^2}{3}\left(\frac{T}{T_{coh}}\right)^2 = A_R T^2 \qquad (13)$$

where

$$A_R = \frac{\rho_0\pi^2}{3}\frac{k_B^2}{\Gamma_L\chi_L}\frac{1}{|J_Q - J_Q^c|^2}.$$

Notice that, *the same coherence temperature*, T_{coh}, appears in the transport properties. In this case, it is the characteristic temperature below which the resistivity varies quadratically with temperature, as appropriate to a FL and that sets the scale for this contribution. Note that there are two characteristic time scales in the problem considered here, namely ξ^z and ξ_{L}^z. While the former vanishes in the local limit the latter may still diverge.

The Kadowaki-Woods ratio [7], $A_R/(C/T)^2$ is given by,

$$\frac{A_R}{(C/T)^2} = \frac{\rho_0 \Gamma_{\mathrm{L}} \chi_{\mathrm{L}}}{3(N k_{\mathrm{B}})^2} \tag{14}$$

which depends on the local parameters, $\Gamma_{\mathrm{L}}\chi_{\mathrm{L}}$, consequently on the nature of the magnetic ion ($4f$, $5f$ or d, for example), but *not* on the distance to the critical point, $|J_Q - J_Q^{\mathrm{c}}|$. From the equation above and the expression for ρ_0 we obtain, $(\Gamma_{\mathrm{L}}\chi_{\mathrm{L}})\left[\frac{J}{nW}\right]^2 = \frac{6.55\times10^{-3}}{n^{2/3}v_0^{1/3}}\frac{A_R}{(C/T)^2}$, with, $n = (N_{\mathrm{c}}/N)$, the average number of conduction electrons per atom and $v_0^{1/3}$, the average atomic radius given in cm. The values of $(\Gamma_{\mathrm{L}}\chi_{\mathrm{L}})(J/W)^2$ obtained from the experimental results and with $n = 1$, are given in Table 1. If we use $(\Gamma_{\mathrm{L}}\chi_{\mathrm{L}}) = 1/2\pi$, the value for $S = 1/2$ [6], we get, $(J/W) = 1.6$, 1.8 and 2.1 for $CeRu_2Si_2$, UPt_3 and $CeCu_6$, respectively, which are too large. On the other hand, extending the Korringa relation [6] for arbitrary spin, *i.e.*, $\Gamma_{\mathrm{L}}\chi_{\mathrm{L}} = 2(\mu/g_J\mu_{\mathrm{B}})^2/3\pi$, where μ is the experimental magnetic moment (in μ_{B}) given in Table 1, we get the values for (J/W) also shown in this table ($n = 1$). These values are in agreement with those expected for non-magnetic heavy fermions [14], although the value of $(J/W)_{\mathrm{c}}$ separating the magnetic from the Fermi liquid ground states in heavy fermions is still unknown. Takimoto and Moriya [15] have also calculated the Kadowaki-Woods ratio for heavy fermions and obtained that, not too close to the QCP, it is nearly independent of $|\delta|$.

Notice that for $T \gg T_{\mathrm{coh}}$, $R(T) \approx \frac{\pi T}{2T_{\mathrm{coh}}}$ and the resistivity varies linearly with temperature. It is given by,

$$\rho(T \gg T_{\mathrm{coh}}) = \rho_0 \Gamma_{\mathrm{L}}\chi_{\mathrm{L}} \frac{\pi}{2}\frac{T}{T_{\mathrm{coh}}}\,.$$

We point out that in the q-dependent, critical regime, *i.e.*, for $q_{\mathrm{c}}\xi \gg 1$, also $\rho = A_R^M T^2$, at low temperatures, but the coefficient $A_R^M \propto |J_Q - J_Q^{\mathrm{c}}|^{-1/2}$ [6] and consequently does not scale as T_{coh}^{-2}, in disagreement with the experimental results in the heavy fermion systems investigated here [1]. Also in the critical regime, the thermal mass, $C/T = 6\pi^2 N k_{\mathrm{B}}^2/Aq_{\mathrm{c}}^2$ [6], as shown before, and the uniform susceptibility, $\chi = N\mu^2/Aq_{\mathrm{c}}^2$ [6] both saturate at non-universal, cut-off dependent values [6], and do not scale as observed experimentally. In this critical regime these quantities are controlled by the *stiffness* A. On the other hand, in the local limit, $q_{\mathrm{c}}\xi \ll 1$, the relevant Fermi liquid parameters are universal since they are independent of the cut-off q_{c} and the stiffness A. They are determined by the coherence temperature (Eqs. (7, 9, 12)), essentially by the distance of the system to the QCP.

It should be clear by now that the spin fluctuation theory of nearly antiferromagnetic systems, in the sub-critical regime $q_{\mathrm{c}}\xi \ll 1$, gives rise to the one-parameter scaling observed in the heavy fermions discussed here. We have obtained, $C/T \propto T_{\mathrm{coh}}^{-1}$, $\chi_0 \propto T_{\mathrm{coh}}^{-1}$, $A_R \propto T_{\mathrm{coh}}^{-2}$ and $h_{\mathrm{c}} \propto T_{\mathrm{coh}}$, as found experimentally in the pressure experiments. The heavy fermions, $CeRu_2Si_2$, $CeCu_6$, UPt_3 and $CeAl_3$ ($P > 1.2$ kbars) are then close, *but not too close*, to the QCP such that they satisfy the condition $q_{\mathrm{c}}\xi \ll 1$ or $q_{\mathrm{c}}/\sqrt{|\delta|} \ll 1/\sqrt{A}$. In the phase diagram of Figure 1, they are close but to the right of the line $(q_{\mathrm{c}}\xi)^{-1} = 1$, shown in this figure. Note that pressure used in the experiments drives these systems further away to the right of this line where the results obtained here are valid.

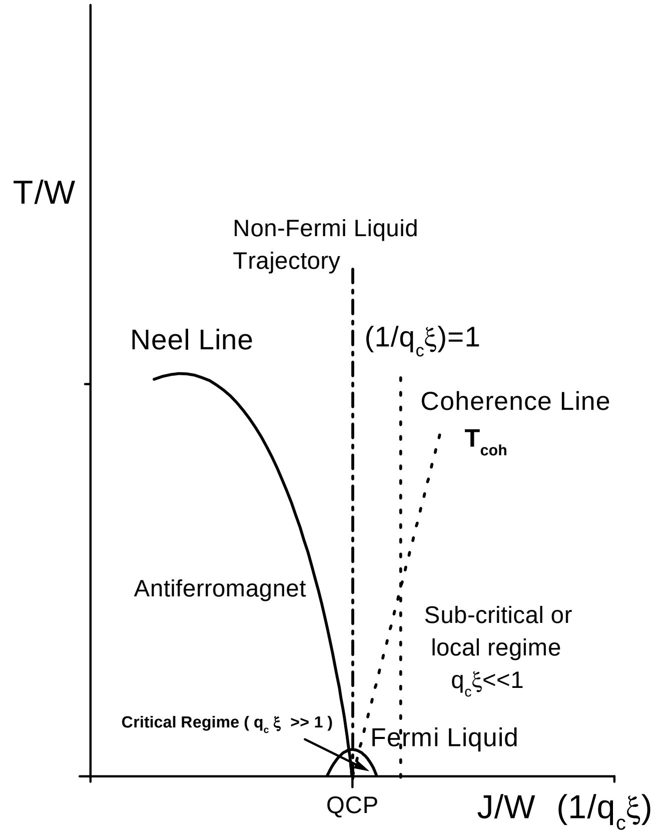

Fig. 1. Phase diagram of heavy fermions. The quantum critical point is located at $(1/q_{\mathrm{c}}\xi) = 0$. Below the coherence line, T_{coh} the system behaves as a Fermi liquid. The line $(1/q_{\mathrm{c}}\xi) = 1$ separates the true critical regime, $(1/q_{\mathrm{c}}\xi) \ll 1$, from the local regime where one-parameter scaling is observed. The crossover along this line is smooth and can be seen as a dimensional crossover from $d = 0$, the local regime, to $d = 3$ (see text). The materials investigated here are to the right of the line $(1/q_{\mathrm{c}}\xi) = 1$.

5 Conclusion

We have shown that the linear term of the specific heat, the uniform susceptibility and the coefficient of the T^2 term of the resistivity of a nearly antiferromagnetic metallic system in the Fermi liquid regime, for $q_{\mathrm{c}}\xi \ll 1$, can be written in terms of the coherence temperature. The condition, $q_c\xi \ll 1$, or $q_c\,(A/|\delta|)^{1/2} \ll 1$, does not imply that the system is far away from the critical point since it can be satisfied for arbitrarily small $|\delta| = |J_Q - J_Q^c|$, for A sufficiently small.

The spin fluctuation theory in the sub-critical regime $q_c\xi \ll 1$ corresponds to a *local interacting model* as the correlation length may become smaller than the distance among spins. Formally, this regime is equivalent to a problem in Euclidean dimension $d = 0$, consistent with the local character of the model in this limit, but with effective dimensionality $d_{\text{eff}} = z = 2$ [4]. In the local model the exponent α, defined through $f_{\text{sf}}^{\text{L}} \propto |\delta|^{2-\alpha}$ for $\delta \to 0$, takes the value $\alpha = 1$ as can be seen from equation (8) with $h = 0$. From the divergence of the characteristic time $\tau_{\text{L}} \propto \xi_{\text{L}}^{z} \propto |\delta|^{-1}$ in equation (11), which yields $\nu z = 1$, we get that the quantum hyperscaling relation [3] $2 - \alpha = \nu(d + z)$ is indeed satisfied for $d = 0$. In spite that spatial correlations are irrelevant in this regime, time fluctuations are critically correlated due to the quantum character of the transition.

In the regime $q_c\xi < 1$, the system obeys a one-parameter scaling, *i.e.*, $C/T \propto T_{\text{coh}}^{-1}$, $\chi_0 \propto T_{\text{coh}}^{-1}$, $A_R \propto T_{\text{coh}}^{-2}$, $h_c = T_{\text{coh}}$, with $T_{\text{coh}} \propto \xi^{-z}$, as we have shown. We can conclude that the heavy fermion systems $CeRu_2Si_2$, $CeCu_6$ and UPt_3, for $P \geq 0$ and $CeAl_3$, for $P > 1.2$ kbars, belong to a region of Doniach's phase diagram where the condition, $q_c\xi < 1$, is satisfied as evidenced by the one-scaling parameter observed in these materials [1,16]. These systems are to the right of the line $(q_c\xi)^{-1} = 1$, in the phase diagram of Figure 1.

As the quantum critical point is further approached, $q_c\xi \to \infty$ and the full q-dependence of the dynamic susceptibility must be taken into account. This critical regime has been described by Takimoto and Moriya [6], but it clearly does not yield the one-parameter scaling obtained in the pressure experiments on the systems above.

Eventually for $|\delta| = 0$, the coherence temperature vanishes and the FL regime is never reached. The thermodynamic properties in this *non-Fermi liquid* regime have been obtained by Moriya [6], Millis [9] and in reference [1], for the case the antiferromagnetic transitions at $T > 0$ are described by non-Gaussian exponents. In fact, although the spin-fluctuation theory gives the exact exponents of the quantum critical point, this is not the case for the finite temperature antiferromagnetic transitions which occur in the region of the Kondo lattice phase diagram for $(J/W) < (J/W)_c$, or $J_Q < J_Q^c$.

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References

1. M.A. Continentino, Phys. Rev. B **57**, 5966 (1998).
2. J. Hertz, Phys. Rev. B **14**, 1165 (1974).
3. M.A. Continentino, G. Japiassu, A. Troper, Phys. Rev. B **39**, 9734 (1989); M.A. Continentino, Phys. Rev. B **47**, 11587 (1993); M. A. Continentino, Phys. Rep. **239**, 179 (1994); M.A. Continentino, Zeitschrift für Physik B **101**, 197 (1996).
4. M.A. Continentino, Physica B **259-261**, 172 (1999).
5. T. Moriya, in *Spin Fluctuations in Itinerant Electron Magnetism* (Springer-Verlag, Berlin Heidelberg, 1985).
6. T. Moriya, T. Takimoto, J. Phys. Soc. Jap. **64**, 960 (1995).
7. K. Kadowaki, S.B. Woods, Solid St. Commun. **58**, 507 (1986).
8. A. Auerbach, K. Levin, Phys. Rev. B **34**, 3524 (1986); P. Coleman, J. Magn. Magn. Mater. **63; 64**, 245 (1987); K. Miyake, T. Matsuura, C.M. Varma, Solid St. Commun. **71**, 1149 (1989).
9. A.J. Millis, Phys. Rev. B **48**, 7183 (1993).
10. C. Pinettes, C. Lacroix, J. Phys.-Cond. Matter **6**, 10093 (1994); C. Lacroix, C. Pinettes, Physica B **206; 207**, 11 (1995).
11. J.D. Thompson, J.M. Lawrence, Lanthanides/Actinides: Physics-II in *Handbook on the Physics and Chemistry of Rare Earths*, edited by K.A. Gschneider Jr, L. Eyring, G.H. Lander, G.R. Choppin (Elsevier Science B.V., 1994), Vol. 19, Chap. 133, p. 383.
12. S. Sachdev, Z. Phys. B **94**, 469 (1994).
13. P. Lederer, D.L. Mills, Phys. Rev. **165**, 837 (1968); A.B. Kaiser, S. Doniach, Int. J. Magn. **1**, 11 (1970).
14. B. Coqblin, *Electronic Structure of Rare-Earth Metals and Alloys: the Magnetic Heavy Rare-Earths* (Academic Press Inc., London, 1977); R. Jullien, M.T. Beal-Monod, B. Coqblin, Phys. Rev. Lett. **30**, 1057 (1973).
15. T. Takimoto, T. Moriya, Solid St. Commun. **99**, 457 (1996).
16. We do not calculate numerically this condition since q_c and A are unknown. It can in fact be relaxed, as the local model already holds for $q_c\xi \lesssim 1$ [1].